

Session 7

Minimum Cost Spanning Tree

This session is about building a minimum cost spanning tree from a connected, undirected weighted graph. A spanning tree connects all vertices without forming a cycle, and the minimum cost spanning tree chooses the set of edges whose total weight is as small as possible.

Objectives

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- Understand the minimum cost spanning tree problem.
- Implement Prim's algorithm and Kruskal's algorithm.
- Compare both algorithms on different graph instances.

Concept

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A spanning tree connects all vertices of a connected, undirected graph without cycles. A minimum cost spanning tree chooses the set of edges with the lowest possible total weight.

Prim's Algorithm

Prim's algorithm grows one tree. It starts from any vertex and repeatedly adds the minimum-weight edge that connects a visited vertex to an unvisited vertex.

Kruskal's Algorithm

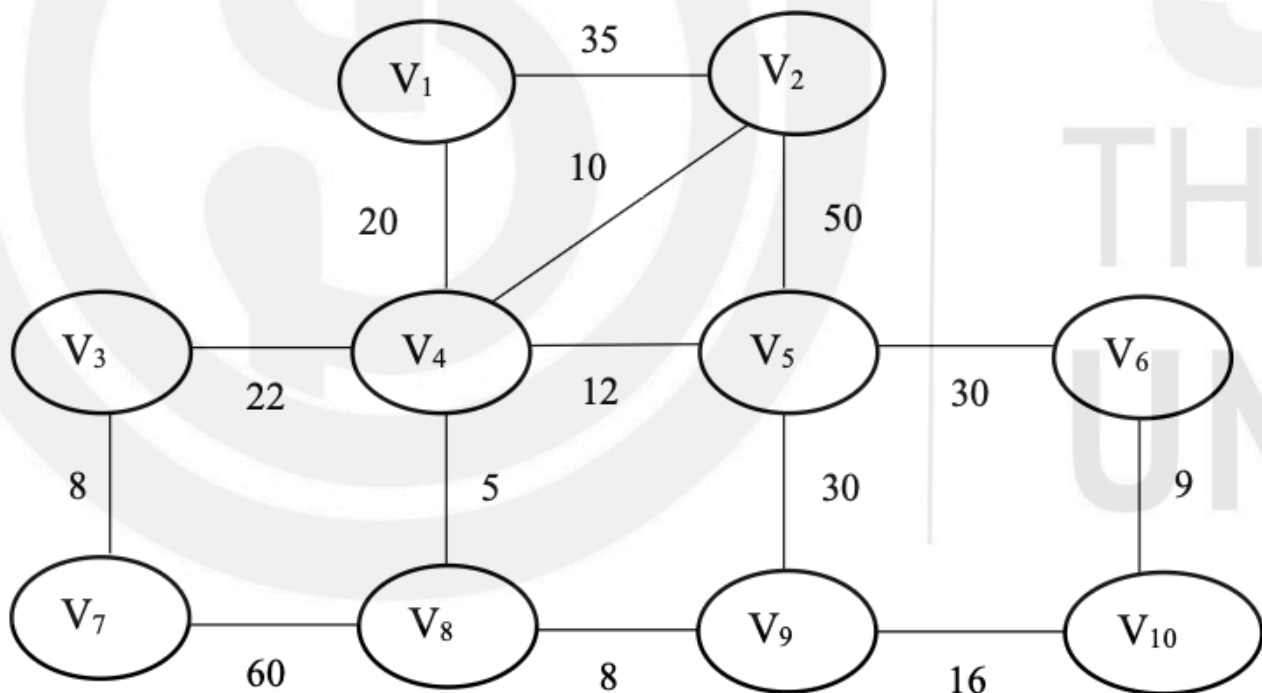
Kruskal's algorithm sorts all edges by weight and repeatedly picks the smallest edge that does not form a cycle.

Question 1

Problem Statement

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Implement Prim's algorithm to find a minimum cost spanning tree(MCST) in the given graph. Show all the processes.



Explanation / Approach

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Start from any vertex and repeatedly choose the minimum-weight edge that connects the current tree to an unvisited vertex. Keep a table of selected edges and running total cost.

Algorithm

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Prim's algorithm is a greedy algorithm that builds the MST incrementally:

- Start with an arbitrary vertex and add it to the MST set.
- Maintain a `key` array storing the minimum weight edge connecting each vertex to the current MST.
- Repeatedly select the vertex with the smallest `key` not yet in MST.
- Add the connecting edge to MST, update keys of adjacent vertices.
- Repeat until all vertices are included.

Space & Time Complexity

Space Complexity: $O(V + E)$ Time Complexity: $O(E \log V)$ with priority queue, $O(V^2)$ with array scan.

Execution Steps

Step	Added	Edge	Weight	Cumulative Cost	MST Edges
0	V1	-	0	0	{}
1	V4	V1-V4	20	20	V1-V4
2	V8	V4-V8	5	25	V1-V4, V4-V8
3	V9	V8-V9	8	33	V1-V4, V4-V8, V8-V9
4	V2	V4-V2	10	43	V1-V4, V4-V8, V8-V9, V4-V2
5	V5	V4-V5	12	55	V1-V4, V4-V8, V8-V9, V4-V2, V4-V5
6	V10	V9-V10	16	71	V1-V4, V4-V8, V8-V9, V4-V2, V4-V5, V9-V10
7	V6	V10-V6	9	80	V1-V4, V4-V8, V8-V9, V4-V2, V4-V5, V9-V10, V10-V6
8	V3	V4-V3	22	102	V1-V4, V4-V8, V8-V9, V4-V2, V4-V5, V9-V10, V10-V6, V4-V3
9	V7	V3-V7	8	110	V1-V4, V4-V8, V8-V9, V4-V2, V4-V5, V9-V10, V10-V6, V4-V3, V3-V7

Final MCST Cost: 110

MST Edges:

V1 → V4 → V8 → V9 → V2 → V5 → V10 → V6 → V3 → V7

Implementation

Write in lab record



Lab record: write one language only. Pick yours once and every page opens on it; the other tabs are the same solution for comparison.

Python

prims-algo.py

```

1 # Prim's Algorithm for Minimum Cost Spanning Tree
2
3 import heapq
4
5
6 class PrimsAlgorithm:
7     def __init__(self):
8         # Adjacency list: vertex → [(neighbor, weight), ...]
9         self.graph = {}
10
11     def add_edge(self, u, v, w):
12         """Add undirected edge between u and v with weight w"""
13         self.graph.setdefault(u, []).append((v, w))
14         self.graph.setdefault(v, []).append((u, w))
15
16     def prim(self, start_vertex):
17         """
18         Execute Prim's algorithm from start_vertex.
19         Returns: mst_edges, total_cost, step_by_step_log
20         """
21         mst_edges = []
22         total_cost = 0
23         visited = set()
24         # Min-heap stores: (weight, from_vertex, to_vertex)
25         heap = [(0, None, start_vertex)]
26         steps = []
27
28         print(f"\n{'=' * 70}")
29         print(f"PRIM'S ALGORITHM - Starting Vertex: {start_vertex}")
30         print(f"{'=' * 70}")
31         print(
32             f"{'Step':<4} | {'Added':<5} | {'Edge':<8} | {'Weight':<6} | {'C"
33         )
34         print(f"{'-' * 70}")
35
36         while heap and len(visited) < len(self.graph):
37             weight, frm, to = heapq.heappop(heap)
38
39             # Skip if already included in MST
40             if to in visited:
41                 continue
42
43             visited.add(to)

```

```
44         if frm is not None:
45             mst_edges.append((frm, to, weight))
46             total_cost += weight
47
48             # Log this step
49             step_info = {
50                 "step": len(steps) + 1,
51                 "added": to,
52                 "edge": f"{frm}-{to}",
53                 "weight": weight,
54                 "cost": total_cost,
55                 "edges": list(mst_edges),
56             }
57             steps.append(step_info)
58
59             # Print formatted row
60             edge_str = ", ".join([f"{u}-{v}" for u, v, _ in mst_edges])
61             print(
62                 f"{step_info['step']:<4} | {to:<5} | {step_info['edge']}>
63             )
64
65             # Explore neighbors
66             for neighbor, w in self.graph.get(to, []):
67                 if neighbor not in visited:
68                     heapq.heappush(heap, (w, to, neighbor))
69
70             print(f"\n{'=' * 70}")
71             print(f"TOTAL MINIMUM COST: {total_cost}")
72             print(f"{'=' * 70}")
73             return mst_edges, total_cost, steps
74
75
76 def main():
77     prims = PrimsAlgorithm()
78
79     # Build the exact graph from the problem image
80     edges = [
81         ("V1", "V2", 35),
82         ("V1", "V4", 20),
83         ("V2", "V4", 10),
84         ("V2", "V5", 50),
85         ("V3", "V4", 22),
86         ("V3", "V7", 8),
87         ("V4", "V5", 12),
```

```
88         ("V4", "V8", 5),
89         ("V5", "V6", 30),
90         ("V5", "V9", 30),
91         ("V6", "V10", 9),
92         ("V7", "V8", 60),
93         ("V8", "V9", 8),
94         ("V9", "V10", 16),
95     ]
96
97     for u, v, w in edges:
98         prims.add_edge(u, v, w)
99
100
101     # Run Prim's starting from V1
102
103     prims.prim("V1")
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Question 2

Problem Statement

■ Write in lab record

Implement Kruskal's algorithm to find a minimum cost spanning tree for the given graph. Show all the processes.

Explanation

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Kruskal's algorithm is a classic greedy algorithm used to discover an MCST. It focuses directly on the edges of the graph. The algorithm operates by selecting the absolute shortest edges from

anywhere in the graph, one by one, and placing them into a growing forest.

The Greedy Choice Strategy:

- Maintain a set of edges that form a forest (where each vertex starts as its own tree).
- Sort all edges in the graph in ascending order of their weights.
- Iterate through the sorted edges and pick the smallest edge.
- Check if adding this edge forms a cycle with the edges already selected.
 - If no cycle is formed, include this edge in the MCST.
 - If a cycle is formed, discard it.
- Terminate when the MCST contains exactly $V - 1$ edges (where V is the total number of vertices).

Cycle Detection via Disjoint-Set (Union-Find)

To efficiently check for cycles, the Disjoint-Set Data Structure is utilized.

- **Find:** Determines which subset a particular element belongs to. If two vertices share the same subset root, connecting them will create a cycle.
- **Union:** Joins two distinct subsets into a single subset when a valid edge is added.

Complexity Analysis

- **Time Complexity:** $O(E \log(E))$ or $O(E \log(V))$, where E is the number of edges and V is the number of vertices. Sorting the edges dominates the computational execution time.
- **Space Complexity:** $O(V + E)$ to maintain the graph data structures and the tracking arrays (`parent` and `rank`) for Union-Find operations.

Implementation

■ Write in lab record

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Lab record: write one language only. Pick yours once and every page opens on it; the other tabs are the same solution for comparison.

Python

kruskal-algo.py

```
1 class Graph:
2     def __init__(self, vertices):
3         self.V = vertices # Total count of nodes
4         self.graph = [] # Default storage array for data streams
5
6     def add_edge(self, u, v, w):
7         """Append an un-directed edge configuration containing weight"""
8         self.graph.append([u, v, w])
9
10    def find(self, parent, i):
11        """Find the root element of element i with path compression technique
12        if parent[i] == i:
13            return i
14        return self.find(parent, parent[i])
15
16    def union(self, parent, rank, x, y):
17        """Perform union of two sets using rank optimization"""
18        xroot = self.find(parent, x)
19        yroot = self.find(parent, y)
20
21        if rank[xroot] < rank[yroot]:
22            parent[xroot] = yroot
23        elif rank[xroot] > rank[yroot]:
24            parent[yroot] = xroot
25        else:
26            parent[yroot] = xroot
27            rank[xroot] += 1
28
29    def kruskal_mcst(self):
30        result = [] # Stores the final built tree
31        i, e = (
32            0,
33            0,
34        ) # i: tracking index for sorted edges, e: tracking count for MCST
35
36        # Step 1: Sort all edges in non-decreasing order of weight
37        self.graph = sorted(self.graph, key=lambda item: item[2])
38
39        parent = []
40        rank = []
41
42        # Create V individual subset components
43        for node in range(self.V):
```

```
44         parent.append(node)
45         rank.append(0)
46
47         # Process elements until the tree contains exactly V-1 components
48         while e < self.V - 1:
49             u, v, w = self.graph[i]
50             i = i + 1
51             x = self.find(parent, u)
52             y = self.find(parent, v)
53
54             # If roots are distinct, no cycle is formed. Accept edge.
55             if x != y:
56                 e = e + 1
57                 result.append([u, v, w])
58                 self.union(parent, rank, x, y)
59
60         # Print out compiled outputs
61         minimum_cost = 0
62         print("Edges in the constructed MCST (Python Implementation):")
63         for u, v, weight in result:
64             minimum_cost += weight
65             print(f"V{u + 1} -- V{v + 1} = {weight}")
66         print(f"Minimum Spanning Tree Cost: {minimum_cost}")
67
68
69 # Driver execution matching our graph data schema
70 if __name__ == "__main__":
71     g = Graph(10)
72     g.add_edge(0, 1, 35) # V1-V2
73     g.add_edge(0, 3, 20) # V1-V4
74     g.add_edge(1, 3, 10) # V2-V4
75     g.add_edge(1, 4, 50) # V2-V5
76     g.add_edge(2, 3, 22) # V3-V4
77     g.add_edge(2, 6, 8) # V3-V7
78     g.add_edge(3, 4, 12) # V4-V5
79     g.add_edge(3, 7, 5) # V4-V8
80     g.add_edge(4, 5, 30) # V5-V6
81     g.add_edge(4, 8, 30) # V5-V9
82     g.add_edge(5, 9, 9) # V6-V10
83     g.add_edge(6, 7, 60) # V7-V8
84     g.add_edge(7, 8, 8) # V8-V9
85     g.add_edge(8, 9, 16) # V9-V10
86
87     g.kruskal_mcst()
```

Sample Output

■ Write in lab record

SH

```
1 Edges in the constructed MCST:
2 V4 -- V8 = 5
3 V3 -- V7 = 8
4 V8 -- V9 = 8
5 V6 -- V10 = 9
6 V2 -- V4 = 10
7 V4 -- V5 = 12
8 V9 -- V10 = 16
9 V1 -- V4 = 20
10 V3 -- V4 = 22
11 Minimum Spanning Tree Cost: 110
```

Question 3

Problem Statement

■ Write in lab record

Analyze the performance of both algorithms on different problem instances and write a brief report.

Explanation / Approach

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This report provides an analytical comparison of Kruskal's and Prim's algorithms for finding a Minimum Cost Spanning Tree (MCST). While both guarantee an optimal minimum spanning tree, their architectural designs perform differently depending on the structural density and representation of the input graph.

Algorithmic Approaches & Data Structures

The core performance differences stem from how each algorithm traverses the graph and what data structures manage its state.

Kruskal's Algorithm

- **Strategy:** Edge-centric greedy approach. It processes the entire graph globally, selecting the lightest available edges regardless of connectivity.
- **Core Mechanisms:**
 - Array sorting or a min-heap to order the edges.
 - **Disjoint-Set (Union-Find)** data structure with path compression and union-by-rank to prevent cycle formation.

Prim's Algorithm

- **Strategy:** Vertex-centric greedy approach. It starts at a single root vertex and grows the tree continuously outward like a single component.
- **Core Mechanisms:**
 - Tracking of structural weights via a fringe/distance array.
 - **Priority Queue (Min-Heap or Fibonacci Heap)** to continually discover the closest unvisited neighbor.

Recommendations

- **Choose Kruskal's Algorithm when:** The problem involves a sparse graph, edge information is already sorted or easily structured, or you are reading from an edge-list representation. It is highly intuitive to implement and has a small memory footprint for standard workloads.
- **Choose Prim's Algorithm when:** The problem involves a dense graph, or you are handling a map matrix layout where vertex boundaries dictate constraints. When optimized with advanced heaps, Prim's offers unparalleled performance metrics for dense network topographies.

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